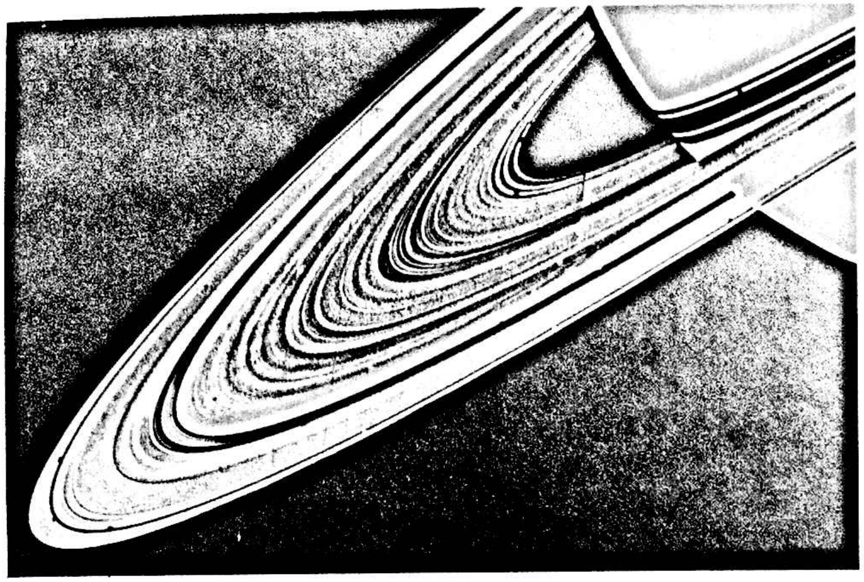


The equatorial rings and ringlets of Saturn are made up of ice particles.

CHAPTER 7
Models of Celestial Motions

You will know something about models of celestial motions if you can:

1. Explain the apparent motions of the planets.
2. Explain the observed motions of fluids at the earth's surface.
3. Describe a geocentric model of the motions of celestial objects.
4. Describe a heliocentric model of the motions of celestial objects.
5. Describe the cyclic changes in kinetic and potential energy of an object in orbit.

In Chapter 6 we gave our attention to the apparent motions of the stars when they are observed through the night and through the year. We also examined the apparent motions of the sun and the moon. To some extent we were repeating the observations of the earliest astronomers. We collected much data, and noticed several patterns of regularity in the observed motions.

Like those early scientists, we can begin to wonder what it all means. What is actually going on out there that can explain what we see? In other words, what "model" of the earth, the sun, the moon, and the stars can account for the observed motions? This chapter will be devoted to the construction of such a model. But first we will need to add certain information about celestial objects called planets—*a word that comes from a Greek word meaning "wanderer."*

MOTIONS OF THE PLANETS

There are several celestial objects that look like stars, but don't quite behave like stars. To the naked eye they seem to be points of light, just as stars do. But unlike stars, they do not keep a fixed position relative to the celestial sphere. As their Greek name tells us, they wander. We will take a detailed look at the wandering of typical planets very shortly.

Another trait of these wanderers is that they seem to have a measurable size. When viewed through a telescope, they become circular objects with a definite diameter. Except for the sun, all stars, even when viewed through the most powerful telescopes, appear only as bright points of light.

The apparent brightness of a planet is not always the same, as is the case with most stars. The brightness increases and decreases in a periodic fashion. Furthermore, when a planet appears brighter, it also appears larger. These observations strongly suggest that the planets approach the earth and then recede from it in a cyclic manner. Our model of the heavens will have to account for those observations.

Some of the planets have markings or features that can be seen with a telescope. These features move across the face of the planet in a constant direction and in a periodic manner. A likely explanation for these observations is that the planets are rotating spheres. This idea will have to be part of our model.

Eastward Motion of the Planets. Table 7-1 (page 94) shows some of the data presently available about our solar system. These data could be corrected in the future. As in almost all areas of science, astronomers are constantly improving their equipment and looking for new ways to gather information. Observations from satellites, free of interference from the earth's atmosphere, are providing more detail and even some new data for astronomers. Some of the most startling data have been received from automated space vehicles designed to travel through the solar system, and in some cases, to continue on into space beyond the planets.

But what can some of the simplest observations tell us about the model we wish to develop? Are the motions of the planets in the sky identical with that of the stars? If not, the difference must become part of our model, and the explanation of those differences must be a part of it also.

All of the planets do, in fact, show the same eastward motion as that of the stars. However, all of the planets *also* show a strange pattern at various times in their motion across our sky. Mars, for example, takes about two years to make a complete eastward trip through the sky and return to the same point relative to the stars. For about three months, however, Mars appears to move *westward* relative to the stars. This apparent "backward" or *retrograde* motion is seen in the movement of all the planets at different times. Furthermore, two of the planets, Mercury and Venus, are peculiar in that they never get very far from where the sun is among the stars. When we finally construct our model of the solar system, these observations must be explained.

	DISTANCE From the Sun in A.U.*	MASS Earth=1	VOLUME Earth=1	PERIOD OF REVOLUTION	NUMBER OF SATELLITES	PERIOD OF ROTATION	DIAMETER Earth=1	AVERAGE DENSITY (g/cm³)	INCLINATION**
MERCURY	0.39	0.05	0.06	88 d	0	58 d	3000 mi 0.38	5.4	0°
VENUS	0.72	0.81	0.88	225 d	0	243 d	7700 mi 0.97	5.3	177°
EARTH	1.0	1.0	1.0	365 d 1.0 y	1	24 h 1.0 d	8000 mi 1.0	5.52	23.5°
MARS	1.52	0.1	0.15	1.9 y	2	1.0 d	4200 mi 0.53	4	25°
JUPITER	5.2	318	1318	11.9 y	15	9.8 h	89,000 mi 11.2	1.3	3°
SATURN	9.5	95	769	29.5 y	20	10.2 h	74,000 mi 9.47	0.7	27°
URANUS	19.2	15	67	84 y	5	10.8 h	30,000 mi 3.75	1.3	98°
NEPTUNE	30.0	17	58	165 y	2	15.7 h	28,000 mi 3.50	1.64	30°
PLUTO	39.4	0.1?	0.06?	248 y	0	6.4 d	3,600 mi 0.45?	4?	?

* 1 A.U. (Astronomical Unit) = 92,955,700 mi = 149,600,000 km, or 1 Earth-Sun distance.
** of the planet's axis to the plane of its orbit.

Table 7-1. Data about the solar system.

Distances of the Planets. One of the data columns in Table 7-1 lists the distance of the planets from the sun. It is also possible to measure the distance to each of the planets from the earth. These measurements yield some interesting results.

First of all, the distance from the earth to each planet varies in a cyclic manner: first near, then far, then near again. Additionally, the time of each planet's approach to the earth varies from the others. For example, the closest approach of Venus to the earth is not at the same time as Jupiter, or any of the other planets.

All the planets do have at least one observation in common. The time of each planet's closest approach to the earth is also at or near the time of its most rapid apparent retrograde motion.

This observation is very important to the development of our model. Telescopic observations allow astronomers to measure the diameters of the planets, as can be seen in Table 7-1. This measurement, however, depends on when it is taken. The diameter measurements for the planets vary in a cyclic manner, just like the distance measurements. As a matter of fact, the largest apparent diameter for each planet is observed at the time of its closest approach to the earth. Remember that this is also the time of the planet's most rapid apparent retrograde motion.

All of the motions and measurements just mentioned form a pattern. Of course, discovering what the pattern means for the purpose of our model may not be easy.

SUMMARY

1. The planets have a generally eastward motion relative to the stars, but periodically move westward for a time.
2. The apparent size of the planets and their distance from the earth vary in a cyclic manner.

TERRESTRIAL OBSERVATIONS

In the previous section all observations of motions were made in the sky; they were *celestial* observations. There is much evidence of motions to be collected on the earth itself; these are *terrestrial* observations. These observations will have to be explained by our model of earth motions.

Foucault Pendulum. If a very long pendulum is suspended and allowed to swing back and forth, you will see it

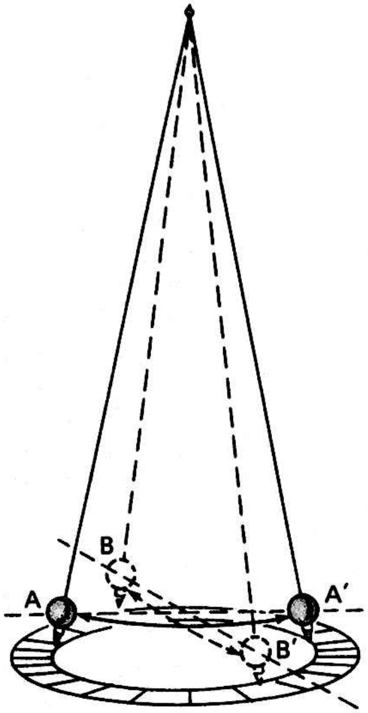


Figure 7-1. Apparent motion of a Foucault pendulum. An observer sees a pendulum swing in the direction *A-A′*. Several hours later the pendulum has changed its direction of swing to the line *B-B′*.

gradually change the direction of its swing (see Figure 7-1). After several hours the pendulum in the illustration will have changed its direction from *A-A*¹ to *B-B*¹. The amount of change can be measured and given as a rate. For example, if the pendulum shifted 10° in 1 hour, the rate of change would be 10°/hr.

The rate of change in the direction of a pendulum on the earth depends on its latitude. At the equator (latitude 0°) the rate of change is zero. The pendulum continues to swing in the same direction. At the poles (latitude 90°) the rate of change is 15°/hr. Notice that this is the same as the apparent rate of rotation of the celestial sphere. At intermediate latitudes, the rate of change is between 0° and 15° per hour. For example, at 42°, it is 10.5°/hr. In the Northern Hemisphere the Foucault pendulum shifts in a clockwise direction when viewed from above. In the Southern Hemisphere the pendulum shifts in a counterclockwise direction.

Coriolis Effect—*Projectile motion.* Projectiles, such as unsteered rockets, ballistic missiles, or shells from long-range cannons, have paths that seem peculiar. These projectiles appear to veer away from the point toward which they were aimed. In the Northern Hemisphere the shift is always to the *right* of the target. In the Southern Hemisphere it is always to the *left* of the target. This change in direction is called the *Coriolis effect.*

Ocean currents. Ocean currents appear to have a pattern of curving toward the right in the Northern Hemisphere and toward the left in the Southern Hemisphere.

Winds. Surface winds around high and low pressure areas have distinctive patterns in the Northern Hemisphere (see Figure 7-2). They have the opposite pattern in the Southern Hemisphere.

SUMMARY

1. A swinging pendulum changes its direction of motion in a manner that is predictable.
2. The path of a freely moving fluid at the surface of the earth or of a projectile appears to undergo a horizontal deflection.

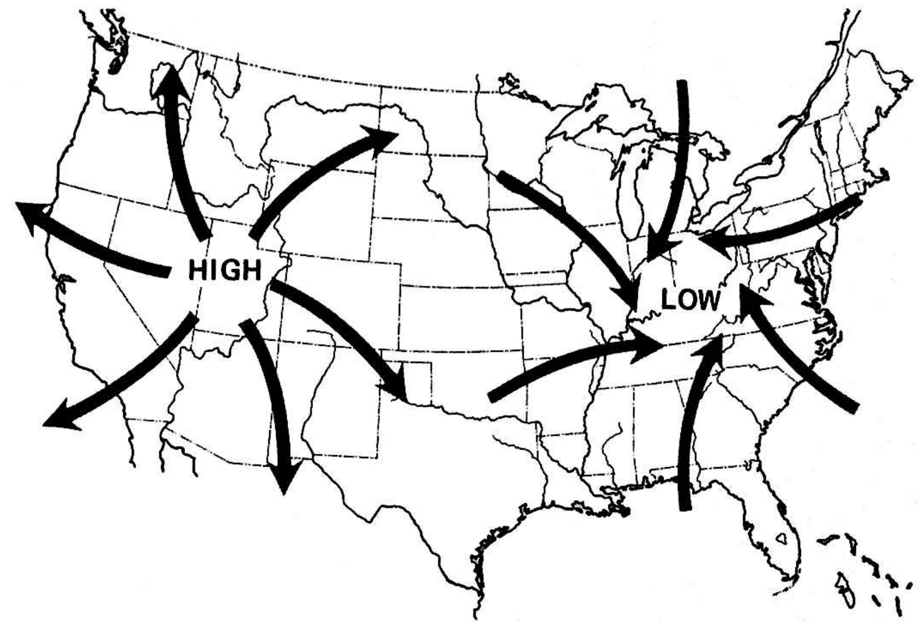


Figure 7-2. The Coriolis effect and winds. Winds blow out from high pressure regions in the atmosphere and in toward low pressure regions. The Coriolis effect deflects these winds to the right in the Northern Hemisphere, with the result that there is a counterclockwise pattern around centers of low pressure and a clockwise pattern around centers of high pressure. These directions are reversed in the Southern Hemisphere.

GEOCENTRIC MODEL OF THE UNIVERSE

"Seeing is believing." It is very human to believe what we see. If we see the sun, the moon, and the stars rise in the east each day, cross the sky, set in the west, and come up again the following day, it is only natural to assume that that is what is actually happening. Therefore, the earliest models of the heavens placed the earth at the center of the universe (or the "world," as it was called). In this model, all the celestial bodies moved around the earth in circles. We call this a geocentric model (*geo-* meaning "earth").

The Celestial Spheres. In the early Greek form of the geocentric model, the celestial bodies were believed to be fixed in transparent spheres. The stars were all in the same sphere, which was the largest or most distant one. The sun, moon, and planets were in smaller spheres. All the spheres revolved around the earth, which was at their center. Differences in the speeds of the spheres accounted for the changing positions of the heavenly bodies among the stars.

The Geocentric Model of Ptolemy. About 2,000 years ago, the Greek astronomer Ptolemy developed a detailed model of the universe based on the idea of revolving spheres. In order to account for the irregular motion of the planets, Ptolemy's model included smaller spheres or circles called *epicycles*. Each planet was located on its own epicycle, and moved uni-

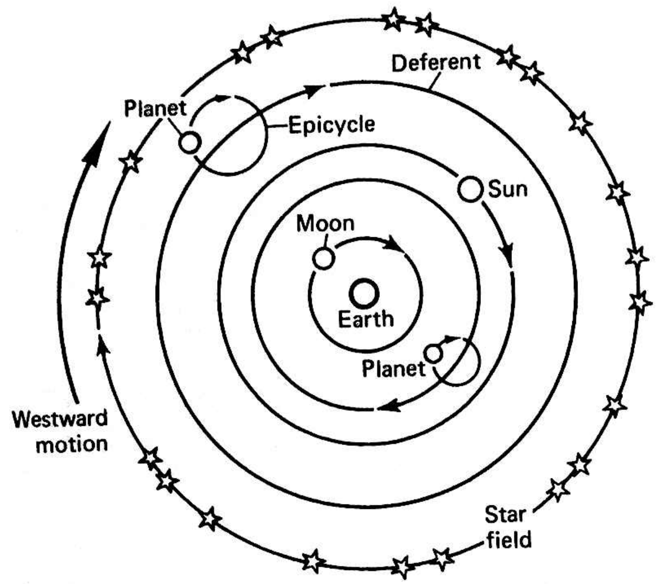


Figure 7-3. Geocentric model of the motions of celestial objects. In this model the earth is stationary. The moon, the sun, and the fixed stars revolve about the earth in circular orbits at different distances and speeds. Each planet revolves in a small circle called an epicycle, while the center of its epicycle moves around the earth along a circle called a deferent. In this diagram you are looking down on the earth from above the North Pole. From this position the celestial objects revolve about the earth in a clockwise (or westward) direction.

formly around the center of the epicycle. Meanwhile, the center of the epicycle was carried uniformly around the earth by one of the spheres.

The model (illustrated in Figure 7-3) can be summarized as follows:

1. The earth is located at the center of the system and does not move.

2. The stars are located on a transparent sphere that rotates once each day from east to west. The axis of rotation extends through the earth at its poles.

3. The sun, the moon, and each planet are carried by separate spheres of different radii. These spheres also rotate from east to west around an axis passing through the earth's poles. However, they rotate at slightly slower speeds than the sphere of the stars. As a result, these bodies have a general eastward drift relative to the stars.

4. Each planet is located on an epicycle that rotates at a fixed rate. The center of the epicycle is carried around the earth at a fixed rate by the planet's sphere, which is called the planet's *deferent.*

5. The deferents and epicycles of the different planets generally rotate at different rates, but the deferents of Mercury and Venus rotate at the same rate as the sun's sphere.

Observations Explained by the Geocentric Model. As we saw in Chapter 6, the stars appear to move as though fixed in a celestial sphere that rotates about the earth once a day. Ptolemy's model therefore explains the daily motion of the stars, because in this model there is an actual sphere of the stars doing just what the celestial sphere appears to do.

The daily motion of the sun, moon, and planets is likewise explained by the daily rotation of their spheres or

deferents. Although this rotation is from east to west, it is somewhat slower than the rotation of the celestial sphere. Therefore these bodies drift slowly eastward relative to the stars—a retrograde motion—as shown in Figure 7-4.

However, it is the center of the planet's epicycle that is shifting uniformly to the east. The planet itself is revolving about that center. Therefore, there are times when its motion along its epicycle will carry it westward faster than its deferent is carrying it eastward. At such times, it will appear to drift westward among the stars—a retrograde motion.

Thus the model explains, in a general way, the features of the apparent motion of the celestial objects. It also explains the changing brightness and apparent size of the planets, since the distance of a planet from the earth varies as it travels along its epicycle.

Difficulties of the Geocentric Model. The test of any model is its success in predicting observations. The chief problem of the Ptolemaic model was that it could not be made to give exact predictions of future positions of the planets. Through the centuries, as more and more data accumulated, the astronomers kept tinkering with the model to make it work better. They added epicycles on top of epicycles to get better agreement with their observations. They moved the center of rotation of the planetary spheres to a point in space away from the earth, and had this point revolve about the earth. By the year 1500, the model had become very complicated, and it still did not work well.

When a scientific model or theory has to be made ever more complicated in order to fit new observations, there is likely to be something basically wrong with it. Scientists and

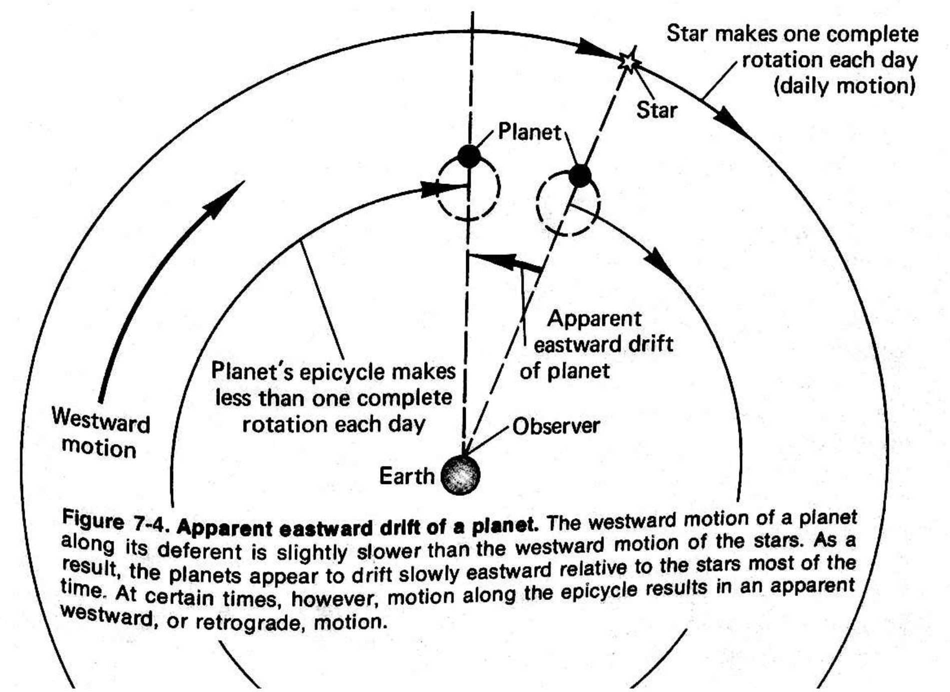


Figure 7-4. Apparent eastward drift of a planet. The westward motion of a planet along its deferent is slightly slower than the westward motion of the stars. As a result, the planets appear to drift slowly eastward relative to the stars most of the time. At certain times, however, motion along the epicycle results in an apparent westward, or retrograde, motion.

philosophers have always believed that the universe is fundamentally simple. A few basic rules, laws, or principles ought to be enough to explain everything—if we can only be clever enough to find them. The increasing complexity of a theory is usually a sign that it is time to look for a fresh idea.

Another difficulty with a model that has a stationary earth is that it does not account for the terrestrial motions described earlier in this chapter. These are modern observations, however, that did not affect the scientific thinking at the time we are referring to (16th century).

SUMMARY

1. A geocentric model of the universe can explain the general features of the apparent motion of the stars, sun, moon, and planets.
2. The geocentric model becomes very complicated when attempts are made to have it predict planetary positions accurately.
3. The geocentric model does not explain terrestrial motions, such as the rotation of a pendulum's direction and the curvature of the paths of projectiles, winds, and ocean currents.

THE HELIOCENTRIC MODEL

In 1543, a new model of the heavens was proposed in a book by the Polish astronomer Copernicus. This model can be summarized as follows:

1. The sun is located at the center of the system and does not move.
2. The stars are located on an unmoving sphere. The sphere is a great distance from the sun.
3. The planets, including the earth, move in circles around the sun. The motions as viewed from the earth are toward the east.
4. The moon moves in a circle around the earth. Its motion is toward the east.
5. The earth rotates on its axis from west to east once each day.

We call this a *heliocentric* model (*helio-* means "sun"). This model is illustrated in Figure 7-5.

Observations Explained by the Heliocentric Model. Like the geocentric model, the Copernican heliocentric model accounts for the daily motion of the celestial bodies, the eastward drift of the sun, moon, and planets through the stars, and the retrograde motion of the planets. It also explains the apparent changes in brightness and diameter of the planets. However, it does all this in a much simpler fashion than the geocentric model of Ptolemy.

1. *Daily motion.* The apparent motion from east to west of all celestial objects around the earth once each day is explained by a single motion of the earth—rotation on its axis from west to east.

2. *Eastward motion of the sun through the stars.* As the earth travels eastward along its path around the sun, the sun each day appears to be in front of a different set of stars, which are slightly to the east of those of the day before. As the earth makes one complete circuit around the sun in one year, the sun appears to make one complete circuit around the celestial sphere along the ecliptic.

3. *Motion of Mercury and Venus.* These two planets travel along orbits inside the earth's orbit around the sun. When they are traveling along the portions of their orbits that pass behind the sun, they appear to move from a point west of the sun, eastward past the sun to a point to the east of the sun. Then they reverse direction and move westward past the sun to a point again to the west of the sun. Since the earth is also moving eastward while this is happening, the two planets share the annual eastward motion of the sun along with their cyclic movement east, and west of the sun (see Figure 7-6).

4. *Motion of the other planets.* All the planets other than Mercury and Venus have orbits outside that of the

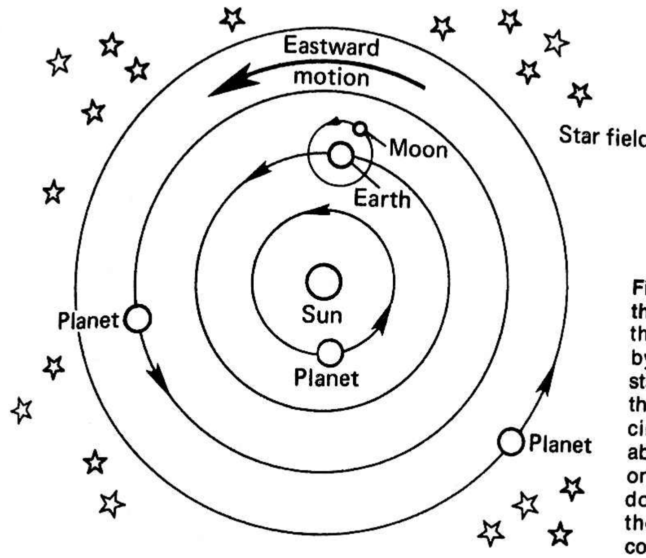
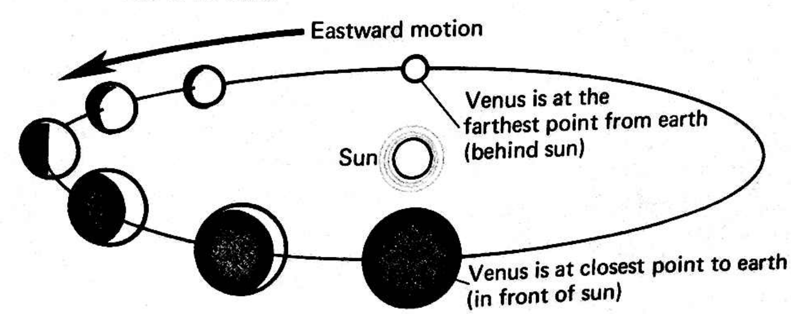


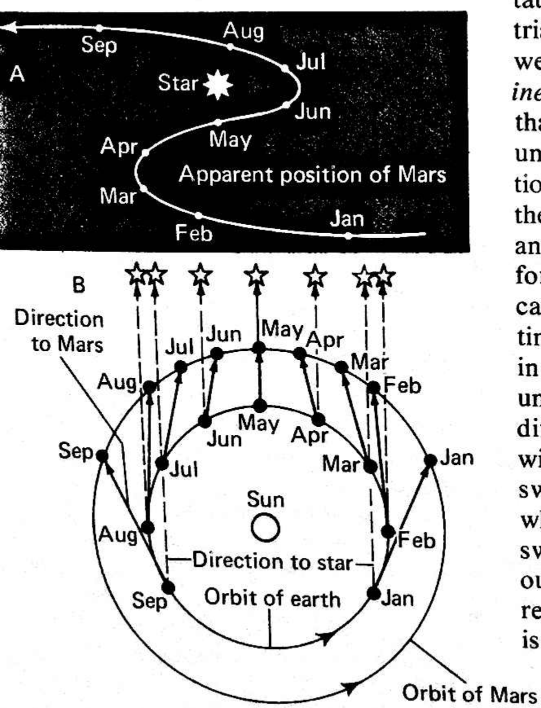
Figure 7-5. Heliocentric model of the motions of celestial objects. In the heliocentric model proposed by Copernicus, the sun and the stars are stationary. The earth and the planets revolve about the sun in circular orbits. The moon revolves about the earth. The earth rotates on its axis once a day. As we look down on the model from above the North Pole, all motions are counterclockwise, or eastward.

Figure 7-6. The apparent changes in position, size, and phase of Venus. In this drawing we are on the earth looking toward the sun. As Venus moves past the sun on the far side of its orbit, we see nearly all of its lighted face, but its apparent size is smallest. As it moves to the east of the sun, its apparent size increases, but we see less of its lighted face. Then as Venus continues along the near side of its orbit, it appears to move westward, to increase in size, and to have a crescent shape. The cycle of changes then reverses as the planet completes it circuit of the sun on the westward side of its orbit.



earth, most of them at great distances from the earth. The planets move eastward along their orbits, the more distant ones moving more slowly. The apparent eastward motion of the planets against the background of the stars is the result of this actual eastward motion. However, the earth is moving around the sun faster than any of these outer planets. Therefore, once each year the earth overtakes each planet and passes it in an eastward direction. During this time, the planet appears to move westward in relation to the stars—a retrograde motion (see Figure 7-7).

Figure 7-7. Apparent motion of Mars. This diagram shows how the position of Mars with respect to a distant star appears to change as viewed from the earth. Because of the star's great distance, the direction from the earth to the star remains practically the same, but the direction to Mars changes. Therefore, the position of Mars with respect to the star appears to change as shown.



5. *Changing brightness and apparent diameter of the planets.* As the earth travels around its orbit, its distance from each planet varies. This accounts for the variations in apparent brightness and diameter in a simple and accurate manner.

6. *Terrestrial motions.* The geocentric model with a stationary earth cannot explain the changing direction of a pendulum or of projectiles and moving fluids on the earth. These motions would have to be treated as something unrelated to the apparent motions of the heavens. On the other hand, the heliocentric model with a rotating earth does account for these terrestrial motions. When one theory can explain two phenomena, while another theory explains only one of them, we tend to prefer the theory that does the more complete job.

In order to understand how the rotation of the earth explains the terrestrial motions that we are considering, we have to refer to the principle of *inertia*. Inertia is a property of matter that causes it to move in a straight line unless a force acts to change its direction. Imagine a pendulum swinging at the North Pole. (see Figure 7-8). At any moment it is swinging back and forth along a certain meridian. Because of its inertia, the pendulum continues to swing in the same direction in space. But the earth is rotating under it. Therefore, as time passes, different meridians come into line with the direction of the pendulum's swing. To an observer on the earth who is turning with it, the direction of swing seems to be changing continuously. Actually, the pendulum's direction in space remains the same. It is the observer who is turning.

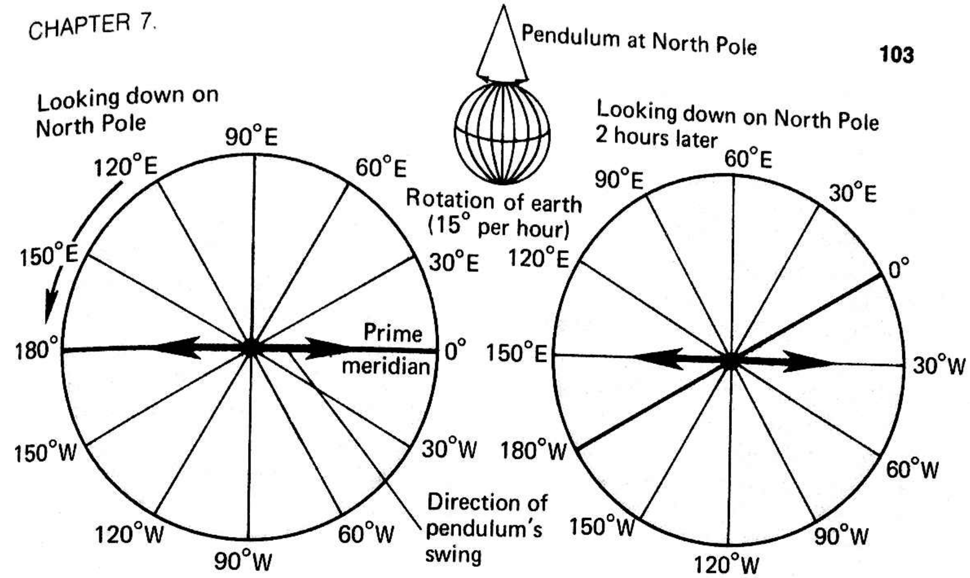


Figure 7-8. Apparent rotation of a swinging pendulum at the North Pole. To an observer at the North Pole, the direction of swing of a pendulum appears to change at the rate of 15°/hr. Actually, it is the earth that is rotating at 15°/hr. while the pendulum continues to swing in the same direction in space.

A pendulum at the pole is the simplest case. It is easy to see why the apparent rate of change of the pendulum's swing is the same as the rate of the earth's rotation, but in the opposite direction. As the pendulum is brought down to lower latitudes, its direction of swing continues to change, but at a slower rate. The rate finally becomes zero at the equator. It is not as easy to understand these more complicated cases. Mathematical analysis, however, shows how the change in direction is related to the earth's rotation.

The change in direction of a projectile is also easiest to understand by imagining the projectile starting at the North Pole and headed south along a particular meridian, as in Figure 7-9. As the projectile moves south in a constant direction, the earth is turning eastward under it. Suppose it travels south to the 60° parallel in 1 hour. In 1 hour, a point on the earth's surface at

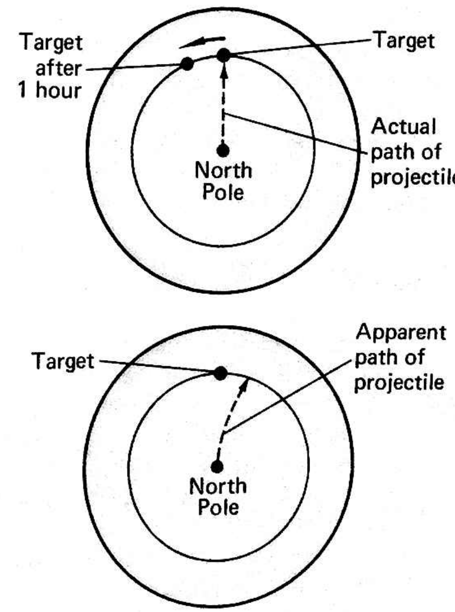


Figure 7-9. Apparent path of a projectile on the earth. A projectile fired from the North Pole toward a target misses the target because the earth turns while the projectile is in motion. To an observer moving with the target, the projectile appears to swerve to the right.

60° latitude moves eastward 839 km. If this point was on the projectile's original meridian (that is, the projectile was aimed at the point), by the time the projectile reached the 60° parallel the point would have moved 839 km to the east. To an observer moving with the earth, the projectile appears to veer to the west. It lands at a point 839 km to the west of the point at which it was aimed. Looking along the path of the projectile from its starting point, it appears to veer to the right.

A projectile appears to veer to the right at any latitude in the Northern Hemisphere, no matter in what direction it is fired. As in the case of the pendulum, the reason is not as obvious as in the simple case we just described. Mathematical analysis is needed to show how the rotation of the earth causes this apparent change in direction.

Difficulties of the Heliocentric Model. The Copernican heliocentric model that we have been examining is much simpler than the Ptolemaic geocentric one, and it works much better. It explains some observations (terrestrial motions) that the Ptolemaic does not. But still the Copernican model has problems. If the earth is revolving around the sun in a circular orbit, the distance to the sun should always be the same. Yet observations of the sun's apparent diameter indicate that the distance to the sun does change by a few percent in a cyclic manner in the course of each year. Furthermore, the sun's apparent speed along the ecliptic varies during the year. This seems to mean that the earth's speed along its orbit varies. It is hard to understand why an object in a circular orbit should speed up and slow down as it goes along.

Likewise, if the moon is revolving about the earth along a circle, its distance from the earth should remain the same at all times and its speed should be constant. Yet its apparent diameter and speed also vary in a cyclic manner.

SUMMARY

1. Apparent motions of celestial objects can be generally explained by a heliocentric model of the universe.
2. Apparent terrestrial motions of the objects are *not* explained by a geocentric model in which the earth is stationary.
3. Apparent motions of celestial objects are generally explained by a heliocentric model in which the earth moves around the sun and rotates around its axis.
4. Apparent terrestrial motions of objects *are* explained by a model in which the earth is rotating.
5. The heliocentric model is simpler than the geocentric model.
6. A heliocentric model with circular orbits does *not* explain the apparent cyclic variations in size and speed of the orbiting bodies.

IMPROVING THE HELIOCENTRIC MODEL

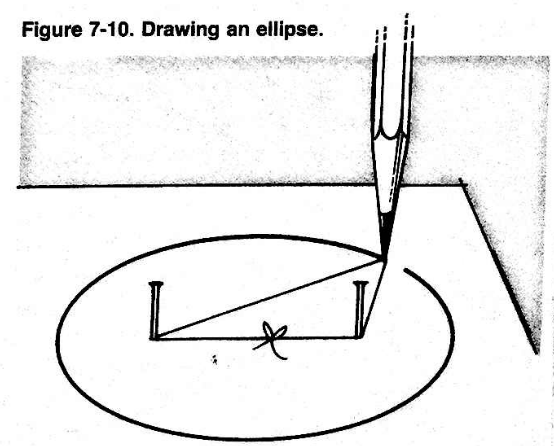
The Copernican model was superior to the Ptolemaic model. It was simpler, and it predicted future positions of the planets much more accurately. But as we have just seen, it did not explain the apparent changes in size and speed of the sun and the moon. And as more accurate observations of the motions of the planets were gathered, it was found that the model did not agree with the detailed data. Let us see how the model was improved.

Tycho Brahe. Tycho Brahe was a Danish astronomer who was born a few years after the publication of Copernicus' theory. Brahe devoted his life to making detailed and accurate records of the positions of the planets and other celestial objects. With the financial support of the king of Denmark, Brahe built the first observatory in history and equipped it with the best instruments that he could devise (the telescope was unknown at the time). With these instruments and painstaking techniques of observation, Brahe accumulated a vast collection of data on the positions of the planets.

Johannes Kepler. Johannes Kepler was Brahe's assistant. He was a brilliant mathematician as well as an astronomer. Kepler believed strongly in the correctness of the heliocentric model. Working with Brahe's tables of observations, he began a long attempt to make the data fit the circular orbits of the Copernican theory. Within a few years, however, he became certain that this could not be done. Kepler now began to try orbits of other shapes. One mathematical shape that he tried was the ellipse, and he discovered that motion of the earth and the planets along elliptical orbits could be brought into good agreement with Brahe's observations.

The Ellipse. An ellipse looks like a flattened circle. The line drawn across the widest part of an ellipse is called its major axis. There are two points along this axis called the *foci* (singular: *focus*) of the ellipse. The sum of the distances between any point on the ellipse and the two foci is the same for all points on the ellipse. Figure 7-10 shows how an ellipse can be drawn. The two pins are at the foci of the ellipse. Since the string has a fixed total length, and the distance between the pins remains the same, the combined length of the two other sides of the triangle also remains the same. Thus we know that the curve satisfies the rule of the ellipse. By changing the distance between the foci, and the length of the string, ellipses of different sizes and different amounts of flattening can be drawn.

Figure 7-10. Drawing an ellipse.



The amount of flattening of an ellipse is measured by its *eccentricity*. The eccentricity of an ellipse is given by the following formula:

$$\text{Eccentricity} = \frac{\text{Distance between foci}}{\text{Length of major axis}}$$

For example, if the foci of an ellipse are 2 cm apart and the length of the major axis is 10 cm, the eccentricity of the ellipse is 2/10 = 0.2.

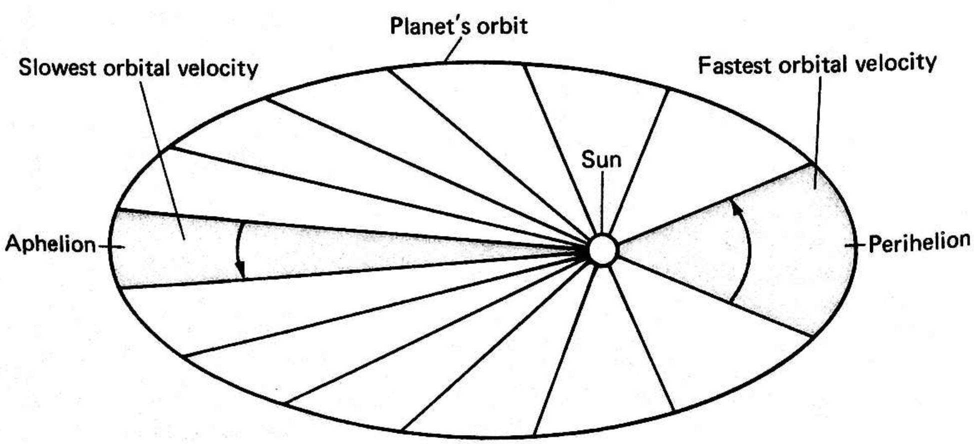
In Kepler's version of the heliocentric model, the orbit of each planet is an ellipse with the sun at one focus. Since the focus is off center, the distance between the planet and the sun varies as the planet moves. The same is true of the earth's orbit around the sun and the moon's orbit around the earth. With ellipses of the right dimensions, Kepler found that the motions of the planets could be brought into excellent agreement with Brahe's observations. The observed variations in apparent size (or distance) of the sun and the moon were also accounted for by the elliptical orbits.

Orbital Speed. Look back at the data in Table 6-2, on page 86. This table shows a cycle of changes in the moon's apparent size. Now let us see how *far* the moon appeared to move in one day when it was smallest (farthest away from the earth). It was smallest from the 14th to the 20th days. Between the 16th day and the 17th day the moon moved from an R.A. of 2 hrs. 34 min. to an R.A. of 3 hrs. 21 min. The difference is about 47 minutes.

What distance did the moon travel when it was largest (closest to the earth)? It was largest on the 32nd day. Between the 31st day and the 32nd day the moon moved from an R.A. of 15 hrs. 43 min. to an R.A. of 16 hrs. 48 min. The difference is about 1 hr. 03 min. So it appears that when the moon is closer to the earth it moves farther. This means it is moving faster, since both observations were made during the period of one day.

Kepler observed the same kind of difference in orbital speed for the planets. This discovery meant that the rate of motion of a planet (its velocity) was always changing. It would be greatest at *perihelion* (closest approach to the sun) and least at *aphelion* (greatest distance from the sun).

Figure 7-11. Kepler's law of equal areas. A line from the sun to a planet will sweep out each of these equal areas in equal periods of time.



Law of Equal Areas. Kepler discovered another law that relates the changing velocity of the planet to the ellipse: Each planet revolves so that a line from the sun to the planet sweeps over equal areas in equal periods of time. Figure 7-11 illustrates the law. This law also applies to any satellite in orbit around any central object.

Law of Planetary Periods. Kepler studied the velocity and distance relationship more closely and was able to express the relationship between them with a mathematical model, or equation. In the equation, T represents the time it takes a planet to make one complete revolution around its orbit. R represents the average distance of the planet from the sun. T_1 and R_1 are values for one planet, T_2 and R_2 are values for another planet. The equation is:

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$$

The average distance of a planet from the sun is equal to half the length of the major axis of its orbit. For the earth, this distance is 149,600,000 km. It is called one astronomical unit (AU) and is often used for measuring distances in the solar system.

SUMMARY

1. In Kepler's modification of the heliocentric model, the earth and the planets move around the sun in elliptical orbits.
2. The sun is at one focus of each orbit.
3. The orbital speed of each planet, including the earth, is greater when the planet is nearer the sun.
4. The variation in orbital speed is such that a line from a planet to the sun sweeps across equal areas in equal times.
5. The greater the average distance of a planet from the sun, the slower its average speed and the longer its period of revolution.

OTHER OBSERVATIONS

In Chapter 6 we mentioned several observations of the sun and the moon that a celestial model must explain. Some of these have already been covered in the discussion of the heliocentric model. Now we will consider others that still need to be accounted for.

The Moon. In Chapter 6 (page 87) we compared the cycle of the moon's apparent position among the stars with the cycle of its phases. We found that one complete cycle through the stars takes about 27⅓ days. One complete cycle of phase changes takes about 29½ days. We noted at the time that our model of the heavens has to account for this difference. By referring to Figure 7-12 we can understand how the heliocentric model does this.

The half of the moon facing the sun is always illuminated by the sun's rays. The other half is dark. As the moon travels around the earth, we see a changing fraction of the lighted half.

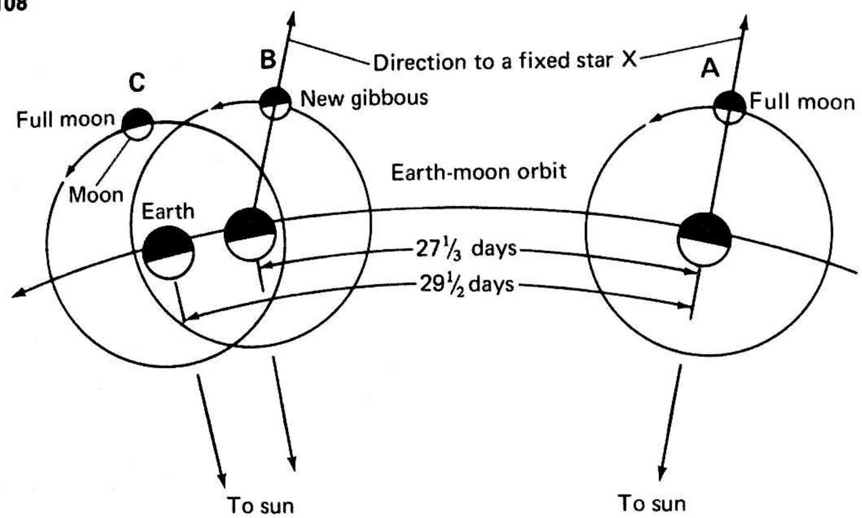


Figure 7-12. One phase cycle of the moon. At position *A*, the moon is full. At position *B*, the moon has made one complete revolution about the earth. However, the moon does not become full until it reaches position *C*, about two days later.

We see a full moon when the moon is on the side of the earth opposite the sun and in line with the earth and the sun. This situation is shown at A in Figure 7-12. (If the sun, earth, and moon were actually in a straight line, the moon would be in the earth's shadow and would be eclipsed. This does happen occasionally, but most of the time the moon is above or below the earth's shadow as it passes behind the earth.)

At B in the diagram, the moon has made a complete circuit of its orbit. It is seen in the same position relative to the stars. But it has not yet become full again. During the $27\frac{1}{3}$ days that this circuit took, the earth moved along its orbit around the sun. The line from the earth to the sun is now in a different direction. It takes a little over 2 days more before the moon is again in line with the earth and the sun, and is therefore again full. By this time it has moved on to a new position relative to the stars. That is, it is well into a second circuit of its orbit.

Changes in Apparent Solar Day. Because the earth moves along its orbit, the cycle of the moon's phases takes longer to complete than the cycle of its position relative to the stars. For similar reasons, the apparent solar day is longer than the sidereal day. The earth has to turn a little more than one complete rotation in order to bring the sun over the same meridian each day, as illustrated in Figure 7-13. When the earth is moving more rapidly along its orbit, it has to turn more to complete a solar day than when it is moving more slowly. Thus the changing speed of the earth is one of the causes for the changing length of the solar day during the year.

The Sun. In the course of a year, the path of the sun across the sky each day changes in a cyclic manner. The altitude of the sun at noon also changes in a cyclic manner. The noon sun can reach an altitude of 90° (be

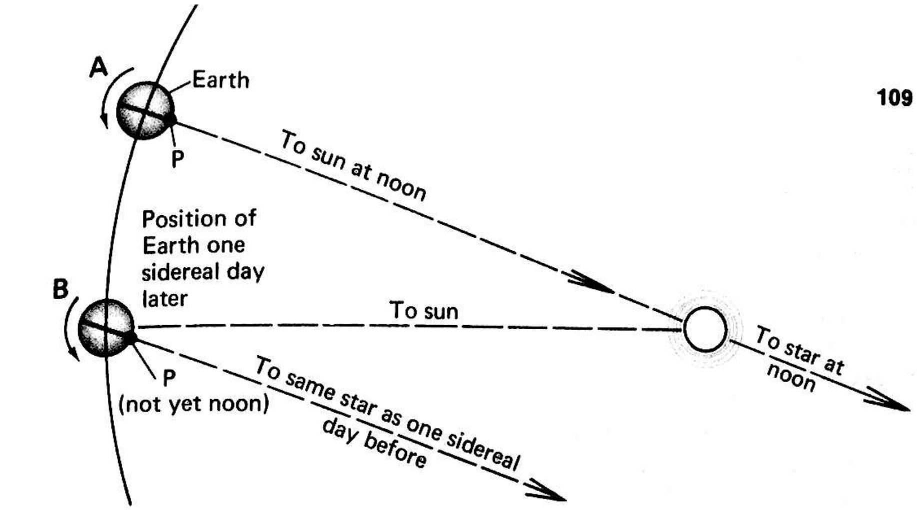


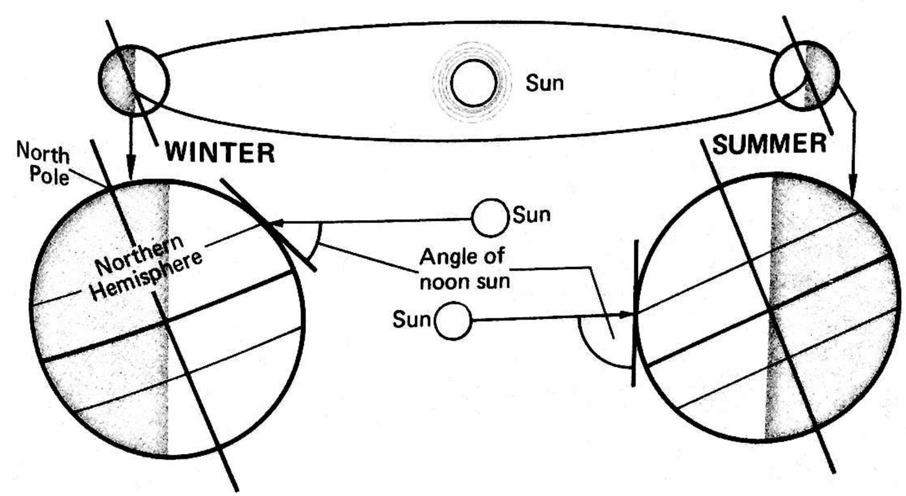
Figure 7-13. The length of a solar day. At position *A*, it is noon at point *P* on the earth. At position *B*, the earth has made one complete rotation relative to the stars. One sidereal day has passed, but it is not yet noon at point *P*. The earth must turn a little more to bring point *P* in line with the sun, when it will be noon again at point *P*. One solar day is therefore longer than one sidereal day.

directly overhead), but this happens only at latitudes between 23½° N and 23½° S.

Our model of earth motions explains these observations by having the axis of the earth tilted at an angle of 23½° to the vertical with respect to the earth's orbit (see Figure 7-14). As the earth travels along its orbit, the axis always points in the same direction in space. That is, the axis points to the same spot among the stars. During one part of the earth's orbit, the North Pole is inclined *away* from the sun. It is winter in the Northern Hemisphere at that time. The angle of the noon sun is its lowest of the year, the arc of the sun's path is shortest, and the duration of daylight is least.

Six months later, the North Pole is still pointing in the same direction in space, but is now inclined toward the

Figure 7-14. Seasonal effect of the tilt of the earth's axis. In the Northern Hemisphere, the angle of the noon sun and the duration of daylight are least in winter, greatest in summer.



sun. It is now summer in the Northern Hemisphere. The noon sun reaches its highest angle, the arc of the sun's path is longest, and the duration of daylight is greatest.

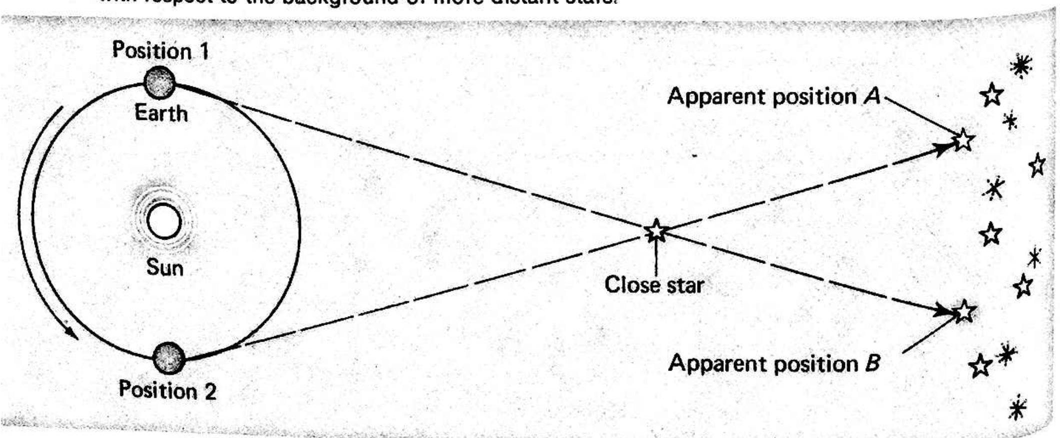
Distances of the Stars. Most early scientists, including Ptolemy and Copernicus, believed that the stars were all on a single sphere at a very great distance from the earth. In their models, the stars were all at the same distance from the earth. They made this assumption because no one had ever seen stars move relative to one another. If the stars were at different distances, and therefore moving on different spheres, it would be expected that they would shift in apparent positions, just as the planets do.

In modern times, it has become possible to observe very small, cyclic shifts in the positions of some stars. According to our modern model of the heavens, these shifts in apparent position occur because of the earth's motion along its orbit around the sun. Figure 7-15 shows the relative positions of the earth, the sun, a "close" star, and a background of more distant stars at two times about 6 months apart. As the earth moves from position 1 to position 2, the apparent position of the close star among the others shifts from position *A* to position *B*.

This apparent shift of one object relative to others as the observer moves is called *parallax*. An example of parallax is easily observed by holding one finger up at arm's length and looking at it first with one eye, then with the other. As you change your view from one eye to the other, your finger appears to change its position against the more distant background.

The diameter of the earth's orbit is about 300,000,000 km. Yet the parallax of the *nearest* star caused by the shift of the earth by this distance is only about 1/5000 degree. This means that even the nearest star is an enormous distance from the earth—actually more than 40,000,000,000,000 km. Astronomers therefore use a special unit of distance—called a *light-year*—for expressing the distances of stars. A light-year is the distance light travels in one *year*, at its speed of 300,000 kilometers per *second*. The nearest star, Alpha Centauri, is more than 4 light-years from us.

Figure 7-15. Parallax of a star. Although all stars are at great distances from the earth, some are farther away than others. As the earth moves from one side of its orbit to the other, a star that is relatively near the earth appears to change position with respect to the background of more distant stars.



Star distances are difficult to imagine. The sun is one star among the billions that make up our *galaxy*—a large group of stars concentrated in one region of space. Our galaxy is estimated to be 100,000 light-years across. Other galaxies are scattered about throughout the universe at even more incredible distances—some measured in hundreds of millions of light-years. Needless to say, such distances are *not* calculated from parallax measurements! They are estimated on the basis of a large number of interconnected astronomical observations, too complex to be summarized here.

SUMMARY

1. The difference between the time for the moon to complete a circuit of the stars and the time to complete a cycle of phases is due to the motion of the earth along its orbit around the sun.
2. The length of the apparent solar day varies because of the changes in the speed of the earth in its orbit.
3. The changes in the sun's daily path across the sky, its noon altitude, and the duration of daylight throughout the year are the result of the tilt of the earth's axis relative to the earth's orbit.
4. Distances to the stars vary over a wide range and are all very large.

FORCE AND ENERGY IN THE CELESTIAL MODEL

In the minds of the ancient Greek philosophers, circular motion was the most natural motion. To them, the circle was the simplest, most "perfect" shape. There was no need to explain *why* the celestial objects moved in circles. Conditions in outer space being perfect, there was no reason for these objects to move in any other way.

This view of the circle as a "natural" shape for celestial orbits continued right up to the time of Copernicus. Uniform motion along circular paths required no explanation. But then Kepler showed that the planets did not move in circular orbits, but in elliptical ones. Besides, they moved at varying speeds, depending on where along the orbit they were at the time. Here was something unexpected and "unnatural." Here was something that needed to be explained. What *causes* the planets to follow elliptical orbits at changing speeds?

Galileo. At about the same time that Kepler was discovering the laws of planetary motion, another brilliant scientist, Galileo, was studying the motions of objects on the earth. In particular, he was seeking the laws of motion for falling bodies. At that time, most scientists accepted Aristotle's belief that heavy objects fall faster than light ones. Galileo did not believe this to be the case. He believed, correctly, that light bodies such as sheets of paper and feathers fall more slowly than heavy objects only because the resistance of the air has a greater relative effect on the light objects.

There is a legend that Galileo dropped two cannon balls of different weights from the Leaning Tower in Pisa to show that they would reach the ground together. Whether or not he did this, it is known that he performed many experiments with balls rolling down inclined planes. From these experiments he learned that the weight of a ball did not affect the time it took to roll down an incline. All balls rolled down the same inclined plane at the same rate. He discovered also that the speed of the rolling balls increased uniformly with time, and that the distance they rolled increased as the square of the time.

From these observations Galileo concluded that objects changed the speed of their motion only when acted upon by a force. If no force acts on a body, it remains at rest if it is at rest, or it continues to move at constant speed in a straight line. This is the principle of inertia that was mentioned earlier in this chapter. It is the key idea for understanding the motion of anything.

Newton. Isaac Newton was born in the year that Galileo died (1642). Newton was one of the world's greatest geniuses, and he made important discoveries in many branches of science and mathematics. He is probably best known for his laws of motion and the theory of gravitation that explains the motion of the planets.

Newton was familiar with Galileo's work on falling bodies. Galileo had stated that a moving body continues moving in a straight line at constant speed unless a force acts on it to change its motion. Newton was familiar also with Kepler's discoveries, and knew that the moon moved around the earth and the planets moved around the sun in elliptical orbits. He wondered what force could cause this constantly changing direction of motion.

According to Newton's own account, he was thinking about this problem while sitting under an apple tree. An apple fell from the tree and hit him on the head. The idea that the earth exerted a force of gravity that caused objects to fall was an old idea. The new idea that came to Newton's mind was the possibility that the force of gravity might extend outward into space—even as far as the moon. If that were so, then the moon was actually falling toward the earth all the time.

Of course, the moon doesn't fall *to* the earth—only *toward* it. Without the force of gravity, the moon would fly away from the earth along a straight-line path. The constant force of gravity keeps pulling the moon away from a straight-line path and into the curved path that it actually follows (see Figure 7-16).

The Law of Gravitation. Newton's idea of gravity means that there is a field of force around the earth. Any object in the field of force will be acted on by the force. This field is a vector field, which means that at every point in space the force has both a *magnitude* and a *direction*. The direction of the earth's gravity field is always toward the center of the earth.

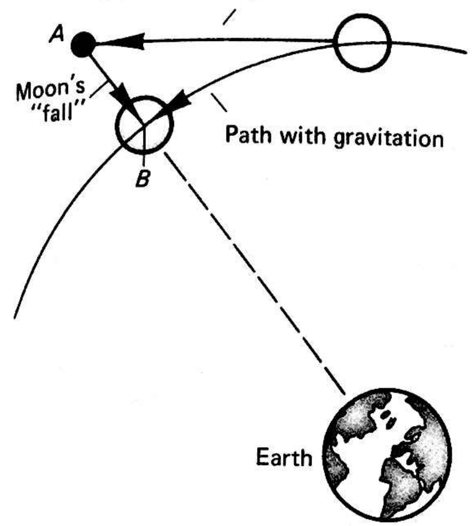


Figure 7-16. How gravitation keeps a moving satellite in its orbit. If the moon stopped moving along its orbit, it would fall to the earth. If there were no gravitation, the moving moon would fly away in a straight line. Because of gravitation, the moving moon neither falls to the earth nor flies away. It continuously "falls" into its orbit.

Newton's calculations led him to conclude that the magnitude of the force of attraction would depend on three factors:

1. The mass of the object (M_1).
2. The mass of the earth (M_2).
3. The distance between the centers of the two masses (R).

Newton suggested a mathematical model (equation) that relates all three factors to the force (F) that is produced. The equation is:

$$F = \frac{G M_1 M_2}{R^2}$$

G is known as the gravitational constant. The mathematical model stated in words is:

> The force of attraction between two masses is directly proportional to the product of the masses, and inversely proportional to the square of the distance between their centers.

Newton suggested that this force extended *throughout the universe.* Any two objects anywhere in the universe would be attracted to each other by the gravitational force between them. This principle is known as the *universal law of gravitation.*

Elliptical Orbits and the Law of Gravitation. Newton's theory of a force of gravitation extending outward through space from every body of matter was a startling one for his time. He dared not publish it without strong evidence for its correctness. The best evidence would be to show that the law of gravitation, combined with the laws of force and motion, explains the elliptical orbits of the moon and the planets. Mathematically, this is very difficult to do, because the distances and therefore the forces are constantly changing as the orbiting body moves. To solve the problem, Newton had to invent a new branch of mathematics, which is called calculus. Using the new method of calculation, Newton was able to show that when a small body revolves about a larger one under the influence of its gravitational field, the orbit of the small body is an ellipse with the large body at one focus. In other words, Newton's model of a gravitational field of force does account for the observed motions of the moon and all the planets.

Energy Transformation and Orbital Motion. If we summarize what we know about objects in motion in orbit, the following facts can be listed:

1. Objects in orbit move in elliptical paths.
2. The *distance* from the orbiting object to the "stationary" object is constantly changing.
3. The speed of the orbiting object

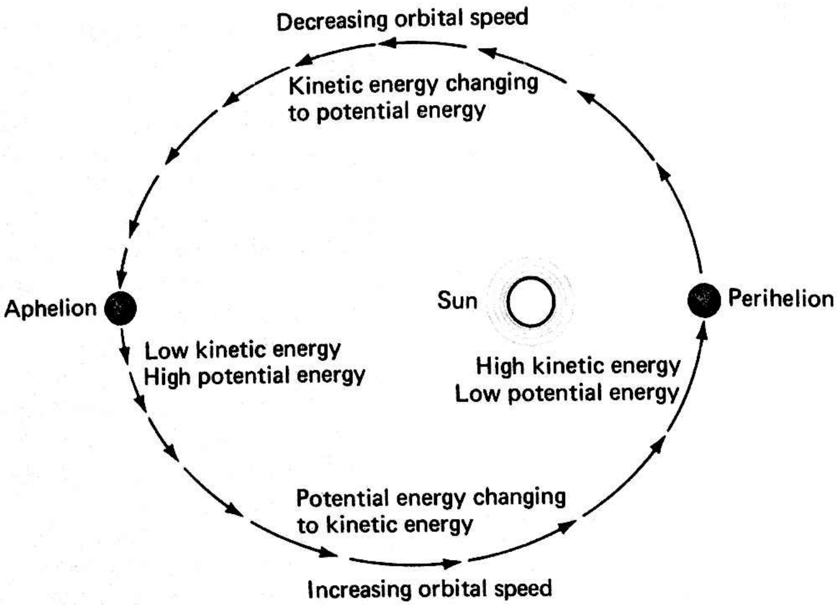


Figure 7-17. Cyclic changes in potential and kinetic energy of a planet in an elliptical orbit.

is constantly changing. It is greatest when the distance is least, and least when the distance is greatest.

The moon's energy of motion, or kinetic energy, is greatest when the moon is closest to the earth, because its speed is greatest at that time. As the moon moves farther away from the earth, it slows down. You might picture the moon as being "higher" above the earth in this position. Its kinetic energy is smaller, but because of its position it has more "stored" energy, or potential energy. As the moon then "falls" back toward the earth, its speed and kinetic energy increase while its potential energy decreases.

The same kind of energy transformation occurs while the earth or any planet travels in orbit around the sun (see Figure 7-17).

SUMMARY

1. A force of attraction exists between any two objects; this is called the force of gravitation.
2. The gravitational force is proportional to the product of the masses and inversely proportional to the square of the distance between their centers.
3. Cyclic energy transformations take place as an object moves along an elliptical orbit. Kinetic energy is changed to potential energy when the object's orbital speed is decreasing, and the reverse transformation occurs when the orbital speed is increasing.

REVIEW QUESTIONS

Group A

1. Is the motion of the planets through the star field uniform?
2. Is the distance to a planet constant, or does it vary? If it does vary, what sort of variation does it show?
3. How do we know that the planets rotate?
4. Why does a long, swinging pendulum appear to change its direction of motion with time?
5. What happens to the path of a freely moving fluid or a projectile at the surface of the earth?
6. Describe the geocentric model of the universe.
7. What observations can be explained relatively simply by the geocentric model?
8. Using the geocentric model, what happens when you try to predict planetary positions accurately?
9. What terrestrial motions cannot be explained by the geocentric model?
10. Describe the heliocentric model of the universe.
11. Which apparent motions of celestial objects can be explained by the heliocentric model?
12. Why is the heliocentric model preferable to the geocentric?
13. In what way did Kepler modify the heliocentric model?
14. Where is the sun in Kepler's modified heliocentric model?
15. When is the orbital speed of a planet greatest? When is it least?
16. State Kepler's law of planetary periods both in words and mathematically.
17. What is the relationship between a planet's distance from the sun, its average speed, and its period of revolution?
18. What causes the difference between the time that it takes the moon to complete a circuit of the stars and the time it takes to complete a cycle of phases?
19. Why does the length of the apparent solar day vary?
20. What causes the changes in the sun's daily path across the sky, its noon altitude, and the duration of daylight throughout the year?
21. What unit is used to describe distances to stars?
22. What force of attraction exists between any two objects?
23. Express Newton's universal law of gravitation both in words and as a mathematical formula.
24. Describe the cyclic energy transformations that take place as an object moves along an elliptical orbit.

Group B

1. What simple observations can you make of a planet such as Mars to show its apparent motions? Explain whether these observations could be done in one evening or would require a longer time.
2. Describe the behavior of a Foucault pendulum at a mid-latitude location such as 42°. How would the pendulum's motion change if the pendulum were carried to a position closer to the poles? Closer to the equator?
3. a. Where is the earth in the geocentric model of the universe?

b. Describe one or more observations of the planets' behavior that is (are) successfully explained by the geocentric model.
c. What feature (or features) of the geocentric model made the model unsatisfactory?

4. a. Where is the earth located in the heliocentric model of celestial motions?
   b. Where is the sun located in the heliocentric model?
   c. How are the moon, planets, and stars located in the heliocentric model?
   d. If a new planet were discovered in the solar system, what prediction, based on the improved heliocentric model, could we make about the motions the new planet would have?
5. What would be the observable effects if the earth's axis were inclined at a greater angle than 23½°? At a smaller angle? (See Table 6-1 and Figure 6-19.)
6. a. What factors affect the force of gravity between two objects?
   b. As the earth orbits the sun, the gravitational force is constantly changing. Describe the pattern of change, relating your description to Figure 7-17, page 114.

REVIEW EXERCISES

1. For an observer at 42°N there are certain stars that never set. Toward which horizon would you look to see these stars? What is the maximum angle above the horizon at which these stars are found? Using Figure 6-3 and the star maps on pages 71 and 72 (Figures 6-4 and 6-5), name some of the stars and constellations that do not set.
2. a. Figure 6-6 on page 73 shows a star trail pattern obtained when a camera was pointed at Polaris. How long was the shutter left open when this photograph was taken? Hint: remember that the earth rotates 360°/24 hours, or 15°/hour.
   b. Does the answer to the previous question depend on which star trail you chose to measure? Explain your answer.
3. a. One of the columns in Table 7-1 shows the distance of the planets from the sun. Explain why these data cannot be used to calculate the distance of the planets from the earth.
   b. Under what conditions is Neptune farthest from the earth? What is this distance in A.U. and in kilometers?
   c. Under what conditions is Neptune closest to the earth? What is this distance in A.U. and in kilometers?

   d. The planet that approaches nearest to Earth would show the greatest change in its position relative to the stars (see page 95). From Table 7-1, which planet would this be?
4. a. One column in Table 7-1 lists the periods of rotation for the planets. Illustrate how these data vary from Mercury to Pluto, either with a generalized graph or by a description. Where are the high points? Where are the low points?
   b. How do the data for the mass, volume, and diameter of the planets compare to their periods of rotation?
   c. Make an overall generalization based on your answers to questions 4a and 4b.
5. Refer to Table 7-1 to answer the following questions.
   a. The primary cause of our seasons is the tilt, or inclination, of the earth's axis. Which planet(s) would have seasons *most like* Earth?
   b. Which planet(s) would have the *least* seasonal change?
   c. Which planet is less dense than water?
6. Make a diagram showing the phases of the earth as seen from the moon for each phase position of the moon shown in Figure 3-4 (page 34).
7. What is the maximum change in altitude of the noon sun at any given location during the course of a year?
8. Describe the sun's daily path at the North Pole on June 21.
9. If you wanted to construct a sundial that could show time in minutes as well as in hours, how much space (in degrees) would there have to be between the minute markings? (Hint: Remember that the sun moves at the rate of 15°/hr.)
10. Suppose we put a satellite in orbit around the sun at a distance of 4 AU from the sun. How many earth years will it take for the satellite to complete one orbit?
11. Four satellites, *A*, *B*, *C*, and *D*, are placed in orbit around the earth. Satellites *A* and *B* are orbiting the earth at exactly the same altitude, but *A* has twice the mass of *B*.
    a. How does the gravitational force of attraction between *A* and the earth compare with the force between *B* and the earth?
    b. How do the periods of revolution of *A* and *B* around the earth compare?
    c. Satellites *C* and *D* have the same mass, but *D* is twice as far from the earth as *C*. How does the force of attraction between *C* and the earth compare with the force between *D* and the earth?
    d. How do the periods of *C* and *D* compare?